**EECE 290 Analog Signal Processing**

**Quiz 3 – April 22, 2017**

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1. Determine *iL*(0+), *vL*(0+), *iC*(0+), and *vC*(0+) (8 grades each), assuming *Kδ*(*t*) = 3 V and initial values *IL*0 = 1 A and *VC*0 = 4 V.

**Solution:** Initially the inductor will act as an open circuit and the capacitor as a short circuit. (40||60) = 24 Ω. Hence, an impulse of  V appears across the inductor and produces a current  A in a direction opposite that of *iL*. It follows that *iL*(0+) =  A. The  V impulse produces a current impulse of  A through the capacitor, resulting in a voltage  V across the capacitor of the same polarity as *vC*. It follows that V.

The circuit at *t* = 0+ is as shown, where (60||12) = 10 Ω. From superposition,  = . Hence, V.

, or  A.

**Version 1:** *K* = 3 Vs: *iL*(0+) = -1 A, *vC*(0+) = 9 V, *vL*(0+) = -6.2 V, *iC*(0+) = -0.38 A.

**Version 2:** *K* = 4.5 Vs: *iL*(0+) = -2 A, *vC*(0+) = 11.5 V, *vL*(0+) = -13.7 V, *iC*(0+) = -0.63 A.

**Version 3:** *K* = 6 Vs: *iL*(0+) = -3 A, *vC*(0+) = 14 V, *vL*(0+) = -21.2 V, *iC*(0+) = -0.88 A.

**Version 4:** *K* = 7.5 Vs: *iL*(0+) = -4 A, *vC*(0+) = 16.5 V, *vL*(0+) = -28.7 V, *iC*(0+) = -1.13 A.

**Version 5:** *K* = 9 Vs: *iL*(0+) = -5 A, *vC*(0+) = 19 V, *vL*(0+) = -36.2 V, *iC*(0+) = -1.38 A.

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1. The switches are closed at *t* = 0 with initial voltages *V*10 = *V*30 = 5 V. Determine: (a) *v*3(*t*) (20 grades), (b) the energy dissipated in the resistor (7 grades), and (c) the energy trapped in the circuit (7 grades).

**Solution:** (a) When the switches are closed, charge is

conserved between *C*1 and *C*2 at the instant of switching. The initial charge on *C*1 is 3*V*10 μC, so that the voltage across the paralleled capacitors at *t* = 0+ is  V across *Ceqp* = 4 μF, and the charge on this capacitor is 3*V*10 μC, as shown. *Ceqs* =  μF with a voltage of   V of the polarity shown, and a charge  μC. The final charge on *C*3 is  μC, and the final voltage is  V. The final charge on *Ceqp* is  V, and the final voltage is  V, which checks.

 µs ≡ 0.1 ms; hence,  =  V, *t* is in ms.

(b) The energy dissipated is the energy stored in *Ceqs*, which is  µJ.

(c) The energy trapped is  µJ.

As a check the sum of these two energies is  µJ. This should equal the energy stored in the 4 μF and 1 µF capacitors at *t* = 0+, which is  μJ.

**Version 1:** *V*10 = 5 V;  V, *Wdiss* = 0.625 µJ, *Wtrap* = 40 µJ.

**Version 2:** *V*10 = 10 V:  V, *Wdiss* = 2.5 µJ, *Wtrap* = 160 µJ.

**Version 3:** *V*10 = 15 V:  V, *Wdiss* = 5.625 µJ, *Wtrap* = 360 µJ.

**Version 4:** *V*10 = 20 V:  V, *Wdiss* = 10 µJ, *Wtrap* = 640 µJ.

**Version 5:** *V*10 = 25 V:  V, *Wdiss* = 15.625 µJ, *Wtrap* = 1 mJ.

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1. *f*(*t*) is described by: *f*(*t*) = ,

0 ≤ *t* ≤1 s, *f*(*t*) = *A*, 1 ≤ *t* ≤ 2 s, *f*(*t*) = , 2 ≤ *t* ≤ 3 s, and *f*(*t*) = 0 elsewhere. *g*(*t*) is a pulse of unit amplitude and 2 s duration. Determine *f*(*t*)\**g*(*t*), for all *t*, by **folding and shifting one of the functions.** Assume *A* = 1.

**Solution:** When *g*(*t*) is folded around the vertical axis and shifted to the right by *t*, the first convolution integral is for 0 ≤ *t* ≤ 1 s:

   ; , .

1 ≤ *t* ≤ 2 s: + ; , .

2 ≤ *t* ≤ 3 s: =

= + *A*  = ; , 

3 ≤ *t* ≤ 4 s:  = = ; ,.

4 ≤ *t* ≤ 5 s:  = = ; , .

*t* ≥ 5 s: .

**Version 1:** *A* = 1.

**Version 2:** *A* = 2.

**Version 3:** *A* = 3.

**Version 4:** *A* = 4.

**Version 5:** *A* = 5.

**Convolution with step functions:** ; *f*(*t*) can also be expressed analytically as: *f*(*t*) =+ *Au*(*t* – 1) – *Au*(*t* – 2) + . The convolution can be divided over time intervals according to the step functions involved.

0 ≤ *t* ≤ 1 s, considering the *u*(*t*) terms:  , as before.

1 ≤ *t* ≤ 2 s, considering *u*(*t*) and *u*(*t* – 1) terms of *f*(*t*):  . Adding this to *y*1(*t*) gives *y*2(*t*) = , as before.

2 ≤ *t* ≤ 3 s, we have to consider terms in *u*(*t* – 2):  + .The first convolution function gives:  . The second convolution function gives: = = . Adding these to *y*2(*t*) gives *y*3(*t*)

3 ≤ *t* ≤ 4 s, we have to consider terms in *u*(*t* – 3):  + . The first convolution function gives: . The second convolution function gives:    . Adding these to *y*3(*t*) gives: .

4 ≤ *t* ≤ 5 s, we have to consider terms in *u*(*t* – 4): . Adding this to *y*4(*t*) gives: .

5 ≤ *t* ≤ 6 s, we have to consider terms in *u*(*t* – 5):   . Adding this to *y*5(*t*) gives *y*6(*t*) = 0.